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CP, T and/or CPT violations in the K^0–$\overline{K^0}$ system

Implications of the KTeV, NA48 and CPLEAR results

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Abstract. The possible violation of the CP, T and/or CPT symmetries in the K^0–$\overline{K^0}$ system is studied from a phenomenological point of view. With this aim, we first introduce parameters which represent the violation of these symmetries in the mixing parameters and decay amplitudes in a convenient and well-defined way and, treating these parameters as small, derive formulas which relate them to the experimentally measured quantities. We then perform numerical analyses, with the aid of the Bell–Steinberger relation, to derive constraints on these symmetry-violating parameters, firstly paying particular attention to the results reported by the KTeV Collaboration and the NA48 Collaboration, and then with the results reported by the CPLEAR Collaboration as well taken into account. A case study, in which either CPT symmetry or T symmetry is assumed, is also carried out. It is demonstrated that the CP and T symmetries are violated definitively at the level of 10^{-4} in 2π decays and presumably at the level of 10^{-3} in the K^0–$\overline{K^0}$ mixing, and that the Bell–Steinberger relation helps us to establish that CP and T violations are definitively present in K^0–$\overline{K^0}$ mixing and to test CPT symmetry to a level of $10^{-4} \sim 10^{-5}$.

1 Introduction

Although, on the one hand, all experimental observations up to now are perfectly consistent with CPT symmetry, and, on the other hand, the standard field theory implies that this symmetry should hold exactly, continued experimental, phenomenological and theoretical studies of this and related symmetries are warranted. In this connection, we like to recall, on the one hand, that CP symmetry is violated only at such a tiny level as 10^{-3} [1,2], while CPT symmetry is tested at best up to a level one order smaller [3–7] and, on the other hand, that some of the premises of the CPT theorem, e.g., locality, are being challenged by, say, the superstring model.

In a series of papers [4–7], we have demonstrated how one may identify or constrain the possible violation of the CP, T and CPT symmetries in the K^0–$\overline{K^0}$ system in a way as phenomenological and comprehensive as possible. For this purpose, we have first introduced parameters which represent the violation of these symmetries in the mixing parameters and the decay amplitudes in a well-defined way, and we related them to the experimentally measured quantities. We have then carried out numerical analyses, with the aid of the Bell–Steinberger relation [8] and with all the available data on the 2π, 3π, $\pi^+\pi^-\gamma$ and $\pi\ell\nu_\ell$ decays used as inputs, to derive constraints to these symmetry-violating parameters. It has been shown among other things that the new results on the asymptotic leptonic asymmetries obtained by the CPLEAR Collaboration [9] allow one for the first time to constrain to some extent the possible CPT violation in the $\pi\ell\nu_\ell$ decay modes[1].

The present work is a continuation of the previous works, which is new particularly as regards the following points:

(1) The new results on $\mathrm{Re}(\varepsilon'/\varepsilon)$, etc., from the Fermilab KTeV and CERN NA48 experiments [11,12], along with CPLEAR's new data [13–16] and the latest version of the data compiled by the Particle Data Group (PDG) [17], are used as inputs.
(2) Particular attention is paid to clarify what can be said without recourse to the Bell–Steinberger relation and what can be said with the aid of this relation.
(3) A case study with either CPT or T symmetry assumed is also carried out.
(4) The relevant decay amplitudes are parametrized in a convenient form, with freedom associated with rephasing

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[1] We afterwards became aware that the workers of the CPLEAR Collaboration themselves [10] had also, by an analysis more or less similar to ours, reached a similar conclusion independently

of both the initial and final states, as discussed explicitly and thoroughly in [4,18,19], taken into account.

This paper is organized as follows. The theoretical framework used to describe the K^0–$\overline{K^0}$ system [20], including the Bell–Steinberger relation, is recapitulated in Sect. 2, and the experimentally measured quantities related to CP violation in the decay modes of interest to us are enumerated in Sect. 3. We then parametrize the mixing parameters and decay amplitudes in a convenient and well-defined way and give conditions imposed by the CP, T and/or CPT symmetries on these parameters in Sect. 4. In Sect. 5, the experimentally measured quantities are expressed in terms of the parameters defined, treating them as being small in first order. In Sect. 6, paying particular attention to the data provided by the KTeV Collaboration and by the NA48 Collaboration, a numerical analysis is performed, while, in Sect. 7, with most of the available experimental data, including those reported by the CPLEAR Collaboration, used as inputs, a more comprehensive numerical analysis is performed. Section 8 is devoted to a case study, in which the case with CPT symmetry assumed and the case with T symmetry assumed are considered separately. The results of the analyses are summarized and some concluding remarks are given in Sect. 9.

2 K^0–$\overline{K^0}$ mixing and the Bell–Steinberger relation

Let $|K^0\rangle$ and $|\overline{K^0}\rangle$ be eigenstates of the strong interaction with strangeness $S=+1$ and -1, related to each other by the (CP), (CPT) and T operations as [4,18,19,21]

$$\begin{aligned}(CP)|K^0\rangle &= \mathrm{e}^{\mathrm{i}\alpha_K}|\overline{K^0}\rangle,\quad (CPT)|K^0\rangle = \mathrm{e}^{\mathrm{i}\beta_K}|\overline{K^0}\rangle,\\ (CP)|\overline{K^0}\rangle &= \mathrm{e}^{-\mathrm{i}\alpha_K}|K^0\rangle,\ (CPT)|\overline{K^0}\rangle = \mathrm{e}^{\mathrm{i}\beta_K}|K^0\rangle,\\ T|K^0\rangle &= \mathrm{e}^{\mathrm{i}(\beta_K-\alpha_K)}|K^0\rangle,\ T|\overline{K^0}\rangle = \mathrm{e}^{\mathrm{i}(\beta_K+\alpha_K)}|\overline{K^0}\rangle.\end{aligned} \tag{2.1}$$

Note here that, given the first two expressions, where α_K and β_K are arbitrary real parameters, the rest follow from the assumptions $(CP)T = T(CP) = (CPT)$, $(CP)^2 = (CPT)^2 = 1$, and from anti-linearity of T and (CPT). When the weak interaction H_{W} is switched on, the K^0 and $\overline{K^0}$ states decay into other states, generically denoted as $|n\rangle$, and get mixed. The time evolution of the arbitrary state

$$|\Psi(t)\rangle = c_1(t)|K_1\rangle + c_2(t)|K_2\rangle,$$

with

$$|K_1\rangle \equiv |K^0\rangle,\quad |K_2\rangle \equiv |\overline{K^0}\rangle,$$

is described by a Schrödinger-like equation [20,22]:

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}|\Psi\rangle = \Lambda|\Psi\rangle,$$

or

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}c_1(t)\\ c_2(t)\end{pmatrix} = \Lambda\begin{pmatrix}c_1(t)\\ c_2(t)\end{pmatrix}. \tag{2.2}$$

The operator or 2×2 matrix Λ may be written as

$$\Lambda \equiv M - \mathrm{i}\Gamma/2, \tag{2.3}$$

with M (the mass matrix) and Γ (the decay or width matrix) given, to second order in H_{W}, by

$$\begin{aligned}M_{ij} &\equiv \langle K_i|M|K_j\rangle\\ &= m_K\delta_{ij} + \langle K_i|H_{\mathrm{W}}|K_j\rangle\\ &\quad + \sum_n P\frac{\langle K_i|H_{\mathrm{W}}|n\rangle\langle n|H_{\mathrm{W}}|K_j\rangle}{m_K - E_n},\end{aligned} \tag{2.4a}$$

$$\begin{aligned}\Gamma_{ij} &\equiv \langle K_i|\Gamma|K_j\rangle\\ &= 2\pi\sum_n \langle K_i|H_{\mathrm{W}}|n\rangle\langle n|H_{\mathrm{W}}|K_j\rangle\delta(m_K - E_n),\end{aligned} \tag{2.4b}$$

where the operator P projects out the principal value. The two eigenstates of Λ and their respective eigenvalues may be written as

$$|K_{\mathrm{S}}\rangle = \frac{1}{\sqrt{|p_{\mathrm{S}}|^2+|q_{\mathrm{S}}|^2}}\left(p_{\mathrm{S}}|K^0\rangle + q_{\mathrm{S}}|\overline{K^0}\rangle\right), \tag{2.5a}$$

$$|K_{\mathrm{L}}\rangle = \frac{1}{\sqrt{|p_{\mathrm{L}}|^2+|q_{\mathrm{L}}|^2}}\left(p_{\mathrm{L}}|K^0\rangle - q_{\mathrm{L}}|\overline{K^0}\rangle\right), \tag{2.5b}$$

$$\lambda_{\mathrm{S}} = m_{\mathrm{S}} - \mathrm{i}\frac{\gamma_{\mathrm{S}}}{2}, \tag{2.6a}$$

$$\lambda_{\mathrm{L}} = m_{\mathrm{L}} - \mathrm{i}\frac{\gamma_{\mathrm{L}}}{2}. \tag{2.6b}$$

$m_{\mathrm{S,L}} = \mathrm{Re}(\lambda_{\mathrm{S,L}})$ and $\gamma_{\mathrm{S,L}} = -2\mathrm{Im}(\lambda_{\mathrm{S,L}})$ are the mass and the total decay width of the $K_{\mathrm{S,L}}$ state, respectively. By definition, $\gamma_{\mathrm{S}} > \gamma_{\mathrm{L}}$ or $\tau_{\mathrm{S}} < \tau_{\mathrm{L}}$ ($\tau_{\mathrm{S,L}} \equiv 1/\gamma_{\mathrm{S,L}}$), and the suffices S and L stand for "short-lived" and "long-lived", respectively. The eigenvalues $\lambda_{\mathrm{S,L}}$ and the ratios of the mixing parameters $q_{\mathrm{S,L}}/p_{\mathrm{S,L}}$ are related to the elements of the mass-width matrix Λ by

$$\lambda_{\mathrm{S,L}} = \pm E + (\Lambda_{11} + \Lambda_{22})/2, \tag{2.7}$$

$$q_{\mathrm{S,L}}/p_{\mathrm{S,L}} = \Lambda_{21}/[E \pm (\Lambda_{11} - \Lambda_{22})/2], \tag{2.8}$$

where

$$E \equiv [\Lambda_{12}\Lambda_{21} + (\Lambda_{11} - \Lambda_{22})^2/4]^{1/2}. \tag{2.9}$$

From the eigenvalue equation of Λ, one may readily derive the well-known Bell–Steinberger relation [8]:

$$\left[\frac{\gamma_{\mathrm{S}}+\gamma_{\mathrm{L}}}{2} - \mathrm{i}(m_{\mathrm{S}} - m_{\mathrm{L}})\right]\langle K_{\mathrm{S}}|K_{\mathrm{L}}\rangle = \langle K_{\mathrm{S}}|\Gamma|K_{\mathrm{L}}\rangle, \tag{2.10}$$

where

$$\langle K_{\mathrm{S}}|\Gamma|K_{\mathrm{L}}\rangle = 2\pi\sum_n \langle K_{\mathrm{S}}|H_{\mathrm{W}}|n\rangle\langle n|H_{\mathrm{W}}|K_{\mathrm{L}}\rangle\delta(m_K - E_n). \tag{2.11}$$

3 Decay modes

The K^0 and $\overline{K^0}$ (or K_{S} and K_{L}) states have many decay modes, among which we are interested in 2π, 3π, $\pi^+\pi^-\gamma$ and the semileptonic modes.

3.1 2π modes

The experimentally measured quantities related to CP violation are η_{+-} and η_{00}, defined by

$$\eta_{+-} \equiv |\eta_{+-}|e^{i\phi_{+-}} \equiv \frac{\langle \pi^+\pi^-, \text{outgoing}|H_W|K_L\rangle}{\langle \pi^+\pi^-, \text{outgoing}|H_W|K_S\rangle}, \quad (3.1a)$$

$$\eta_{00} \equiv |\eta_{00}|e^{i\phi_{00}} \equiv \frac{\langle \pi^0\pi^0, \text{outgoing}|H_W|K_L\rangle}{\langle \pi^0\pi^0, \text{outgoing}|H_W|K_S\rangle}. \quad (3.1b)$$

Defining

$$\omega \equiv \frac{\langle (2\pi)_2|H_W|K_S\rangle}{\langle (2\pi)_0|H_W|K_S\rangle}, \quad (3.2)$$

$$\eta_I \equiv |\eta_I|e^{i\phi_I} \equiv \frac{\langle (2\pi)_I|H_W|K_L\rangle}{\langle (2\pi)_I|H_W|K_S\rangle}, \quad (3.3)$$

where $I = 1$ or 2 stands for the isospin of the 2π states, one gets

$$\eta_{+-} = \frac{\eta_0 + \eta_2\omega'}{1+\omega'}, \quad (3.4a)$$

$$\eta_{00} = \frac{\eta_0 - 2\eta_2\omega'}{1-2\omega'}, \quad (3.4b)$$

where

$$\omega' \equiv \frac{1}{\sqrt{2}}\omega e^{i(\delta_2-\delta_0)}, \quad (3.5)$$

δ_I being the S-wave $\pi\pi$ scattering phase shift for the isospin I state at an energy of the rest mass of K^0. ω is a measure of the deviation from the $\Delta I = 1/2$ rule, and may be inferred, for example, from

$$r \equiv \frac{\gamma_S(\pi^+\pi^-) - 2\gamma_S(\pi^0\pi^0)}{\gamma_S(\pi^+\pi^-) + \gamma_S(\pi^0\pi^0)} = \frac{4\text{Re}(\omega') - 2|\omega'|^2}{1+2|\omega'|^2}. \quad (3.6)$$

Here and in the following, $\gamma_{S,L}(n)$ denotes the partial width for $K_{S,L}$ to decay into the final state $|n\rangle$.

3.2 3π and $\pi^+\pi^-\gamma$ modes

The experimentally measured quantities are

$$\eta_{+-0} = \frac{\langle \pi^+\pi^-\pi^0, \text{outgoing}|H_W|K_S\rangle}{\langle \pi^+\pi^-\pi^0, \text{outgoing}|H_W|K_L\rangle}, \quad (3.7a)$$

$$\eta_{000} = \frac{\langle \pi^0\pi^0\pi^0, \text{outgoing}|H_W|K_S\rangle}{\langle \pi^0\pi^0\pi^0, \text{outgoing}|H_W|K_L\rangle}, \quad (3.7b)$$

$$\eta_{+-\gamma} = \frac{\langle \pi^+\pi^-\gamma, \text{outgoing}|H_W|K_L\rangle}{\langle \pi^+\pi^-\gamma, \text{outgoing}|H_W|K_S\rangle}. \quad (3.8)$$

We shall treat the 3π ($\pi^+\pi^-\gamma$) states as purely CP-odd (CP-even).

3.3 Semileptonic modes

The well-measured time-independent asymmetry parameter related to CP violation in semileptonic decay modes is

$$d_L^\ell = \frac{\gamma_L(\pi^-\ell^+\nu_\ell) - \gamma_L(\pi^+\ell^-\overline{\nu}_\ell)}{\gamma_L(\pi^-\ell^+\nu_\ell) + \gamma_L(\pi^+\ell^-\overline{\nu}_\ell)}, \quad (3.9)$$

where $\ell = e$ or μ. The CPLEAR Collaboration [9,14–16] have furthermore for the first time measured two kinds of time-dependent asymmetry parameters:

$$d_1^\ell(t) = \frac{|\langle \ell^+|H_W|\overline{K^0}(t)\rangle|^2 - |\langle \ell^-|H_W|K^0(t)\rangle|^2}{|\langle \ell^+|H_W|\overline{K^0}(t)\rangle|^2 + |\langle \ell^-|H_W|K^0(t)\rangle|^2}, \quad (3.10a)$$

$$d_2^\ell(t) = \frac{|\langle \ell^-|H_W|\overline{K^0}(t)\rangle|^2 - |\langle \ell^+|H_W|K^0(t)\rangle|^2}{|\langle \ell^-|H_W|\overline{K^0}(t)\rangle|^2 + |\langle \ell^+|H_W|K^0(t)\rangle|^2}, \quad (3.10b)$$

where $|\ell^+\rangle = |\pi^-\ell^+\nu_\ell\rangle$ and $|\ell^-\rangle = |\pi^+\ell^-\overline{\nu}_\ell\rangle$.

4 Parametrization and conditions imposed by CP, T and CPT symmetries

We shall parametrize the ratios of the mixing parameters q_S/p_S and q_L/p_L as

$$\frac{q_S}{p_S} = e^{i\alpha_K}\frac{1-\varepsilon_S}{1+\varepsilon_S}, \quad \frac{q_L}{p_L} = e^{i\alpha_K}\frac{1-\varepsilon_L}{1+\varepsilon_L}, \quad (4.1)$$

and $\varepsilon_{S,L}$ as

$$\varepsilon_{S,L} = \varepsilon \pm \delta. \quad (4.2)$$

From (2.7), (2.8) and (2.9), treating ε and δ as small parameters, one may derive [4]

$$\Delta m \simeq 2\text{Re}(M_{12}e^{i\alpha_K}), \quad (4.3a)$$

$$\Delta\gamma \simeq 2\text{Re}(\Gamma_{12}e^{i\alpha_K}), \quad (4.3b)$$

$$\varepsilon \simeq (\Lambda_{12}e^{i\alpha_K} - \Lambda_{21}e^{-i\alpha_K})/2\Delta\lambda, \quad (4.4a)$$

$$\delta \simeq (\Lambda_{11} - \Lambda_{22})/2\Delta\lambda, \quad (4.4b)$$

from which it follows that [4,5]

$$\varepsilon_\parallel \equiv \text{Re}[\varepsilon\exp(-i\phi_{SW})] \simeq \frac{-2\text{Im}(M_{12}e^{i\alpha_K})}{\sqrt{(\gamma_S-\gamma_L)^2 + 4(\Delta m)^2}}, \quad (4.5a)$$

$$\varepsilon_\perp \equiv \text{Im}[\varepsilon\exp(-i\phi_{SW})] \simeq \frac{\text{Im}(\Gamma_{12}e^{i\alpha_K})}{\sqrt{(\gamma_S-\gamma_L)^2 + 4(\Delta m)^2}}, \quad (4.5b)$$

$$\delta_\parallel \equiv \text{Re}[\delta\exp(-i\phi_{SW})] \simeq \frac{(\Gamma_{11}-\Gamma_{22})}{2\sqrt{(\gamma_S-\gamma_L)^2 + 4(\Delta m)^2}}, \quad (4.6a)$$

$$\delta_\perp \equiv \text{Im}[\delta\exp(-i\phi_{SW})] \simeq \frac{(M_{11}-M_{22})}{\sqrt{(\gamma_S-\gamma_L)^2 + 4(\Delta m)^2}}, \quad (4.6b)$$

where

$$\Delta m \equiv m_S - m_L, \quad \Delta\gamma \equiv \gamma_S - \gamma_L, \quad \Delta\lambda \equiv \lambda_S - \lambda_L, \quad (4.7a)$$

$$\phi_{SW} \equiv \tan^{-1}\left(\frac{-2\Delta m}{\Delta\gamma}\right). \quad (4.7b)$$

$\phi_{\rm SW}$ is often called the superweak phase.

Paying particular attention to the 2π and semileptonic decay modes, we shall parametrize the amplitudes for K^0 and $\overline{K^0}$ to decay into $|(2\pi)_I\rangle$ as

$$\langle(2\pi)_I|H_{\rm W}|K^0\rangle = F_I(1+\varepsilon_I){\rm e}^{{\rm i}\alpha_K/2}, \quad (4.8a)$$
$$\langle(2\pi)_I|H_{\rm W}|\overline{K^0}\rangle = F_I(1-\varepsilon_I){\rm e}^{-{\rm i}\alpha_K/2}, \quad (4.8b)$$

and the amplitudes for K^0 and $\overline{K^0}$ to decay into $|\ell^+\rangle$ and $|\ell^-\rangle$ as

$$\langle\ell^+|H_{\rm W}|K^0\rangle = F_\ell(1+\varepsilon_\ell){\rm e}^{{\rm i}\alpha_K/2}, \quad (4.9a)$$
$$\langle\ell^-|H_{\rm W}|\overline{K^0}\rangle = F_\ell(1-\varepsilon_\ell){\rm e}^{-{\rm i}\alpha_K/2}, \quad (4.9b)$$
$$\langle\ell^+|H_{\rm W}|\overline{K^0}\rangle = x_{\ell+}F_\ell(1+\varepsilon_\ell){\rm e}^{-{\rm i}\alpha_K/2}, \quad (4.9c)$$
$$\langle\ell^-|H_{\rm W}|K^0\rangle = x^*_{\ell-}F_\ell(1-\varepsilon_\ell){\rm e}^{{\rm i}\alpha_K/2}. \quad (4.9d)$$

$x_{\ell+}$ and $x_{\ell-}$, which measure violation of the $\Delta S = \Delta Q$ rule, will further be parametrized as

$$x_{\ell+} = x_\ell^{(+)} + x_\ell^{(-)}, \qquad x_{\ell-} = x_\ell^{(+)} - x_\ell^{(-)}. \quad (4.10)$$

Our amplitude parameters F_I, ε_I, F_ℓ, ε_ℓ, $x_\ell^{(+)}$ and $x_\ell^{(-)}$, and our mixing parameters ε and δ as well, are all invariant with respect to rephasing of $|K^0\rangle$ and $|\overline{K^0}\rangle$,

$$|K^0\rangle \to |K^0\rangle' = |K^0\rangle{\rm e}^{-{\rm i}\xi_K},$$
$$|\overline{K^0}\rangle \to |\overline{K^0}\rangle' = |\overline{K^0}\rangle{\rm e}^{{\rm i}\xi_K}, \quad (4.11)$$

in spite of α_K itself not being invariant with respect to this rephasing [4,18]. F_I, ε_I, F_ℓ and ε_ℓ are, however, not invariant with respect to rephasing of the final states [7, 19],

$$|(2\pi)_I\rangle \to |(2\pi)_I\rangle' = |(2\pi)_I\rangle{\rm e}^{-{\rm i}\xi_I}, \quad (4.12a)$$
$$|\ell^+\rangle \to |\ell^+\rangle' = |\ell^+\rangle{\rm e}^{-{\rm i}\xi_{\ell+}},$$
$$|\ell^-\rangle \to |\ell^-\rangle' = |\ell^-\rangle{\rm e}^{-{\rm i}\xi_{\ell-}}, \quad (4.12b)$$

nor are the relative CP and CPT phases α_ℓ, β_I and β_ℓ defined in such a way that

$$CP|\ell^+\rangle = {\rm e}^{{\rm i}\alpha_\ell}|\ell^-\rangle, \quad (4.13a)$$
$$CPT|(2\pi)_I\rangle = {\rm e}^{{\rm i}\beta_I}|(2\pi)_I\rangle,$$
$$CPT|\ell^+\rangle = {\rm e}^{{\rm i}\beta_\ell}|\ell^-\rangle. \quad (4.13b)$$

One may convince oneself [4,18,19] that the freedom associated with the choice of ξ_I, $\xi_{\ell+}+\xi_{\ell-}$ and $\xi_{\ell+}-\xi_{\ell-}$ allows one, without loss of generality, to take[2]

$${\rm Im}(F_I) = 0, \quad {\rm Im}(F_\ell) = 0, \quad {\rm Im}(\varepsilon_\ell) = 0, \quad (4.14)$$

respectively, and that the CP, T and CPT symmetries impose such conditions as

$$\begin{aligned} CP \text{ symmetry} &: \ \varepsilon = 0,\ \delta = 0,\ \varepsilon_I = 0,\ {\rm Re}(\varepsilon_\ell) = 0,\\ &\quad {\rm Im}(x_\ell^{(+)}) = 0,\ {\rm Re}(x_\ell^{(-)}) = 0\\ T \text{ symmetry} &: \ \varepsilon = 0,\ {\rm Im}(\varepsilon_I) = 0,\ {\rm Im}(x_\ell^{(+)}) = 0,\\ &\quad {\rm Im}(x_\ell^{(-)}) = 0;\\ CPT \text{ symmetry} &: \ \delta = 0,\ {\rm Re}(\varepsilon_I) = 0,\ {\rm Re}(\varepsilon_\ell) = 0,\\ &\quad {\rm Re}(x_\ell^{(-)}) = 0,\ {\rm Im}(x_\ell^{(-)}) = 0. \end{aligned} \quad (4.15)$$

Among these parameters, ε and δ will be referred to as indirect parameters and the rest as direct parameters[3].

5 Formulas relevant for the numerical analysis

We shall adopt a phase convention which gives (4.14). The observed or expected smallness of the violation of the CP, T and CPT symmetries and of the $\Delta I = 1/2$ and $\Delta Q = \Delta S$ rules allows us to treat all our parameters, ε, δ, ε_I, ε_ℓ, $x_\ell^{(+)}$, $x_\ell^{(-)}$ as well as ω' as small[4], and, from (3.2), (3.3), (3.4a,b), (3.6), (3.9) and (3.10a,b), one finds, to the leading order in these small parameters,

$$\omega \simeq {\rm Re}(F_2)/{\rm Re}(F_0), \quad (5.1)$$

$$\eta_I \simeq \varepsilon - \delta + \varepsilon_I, \quad (5.2)$$

$$\eta_{+-} \simeq \eta_0 + \varepsilon', \quad (5.3a)$$
$$\eta_{00} \simeq \eta_0 - 2\varepsilon', \quad (5.3b)$$

$$r \simeq 4{\rm Re}(\omega'), \quad (5.4)$$

$$d_{\rm L}^\ell \simeq 2{\rm Re}(\varepsilon-\delta) + 2{\rm Re}(\varepsilon_\ell - x_\ell^{(-)}), \quad (5.5)$$

$$d_1^\ell(t \gg \tau_{\rm S}) \simeq 4{\rm Re}(\varepsilon) + 2{\rm Re}(\varepsilon_\ell - x_\ell^{(-)}), \quad (5.6a)$$
$$d_2^\ell(t \gg \tau_{\rm S}) \simeq 4{\rm Re}(\delta) - 2{\rm Re}(\varepsilon_\ell - x_\ell^{(-)}), \quad (5.6b)$$

where

$$\varepsilon' \equiv (\eta_2 - \eta_0)\omega'. \quad (5.7)$$

Note that $d_{\rm L}^\ell$, $d_1^\ell(t \gg \tau_{\rm S})$ and $d_2^\ell(t \gg \tau_{\rm S})$ are not independent:

$$d_{\rm L}^\ell \simeq [d_1^\ell(t \gg \tau_{\rm S}) - d_2^\ell(t \gg \tau_{\rm S})]/2. \quad (5.8)$$

From (5.3a,b), it follows that

$$\eta_0 \simeq (2/3)\eta_{+-}(1 + (1/2)|\eta_{00}/\eta_{+-}|{\rm e}^{{\rm i}\Delta\phi}), \quad (5.9)$$

and, treating $|\varepsilon'/\eta_0|$ as a small quantity, which is justifiable empirically (see below), one further obtains

$$\eta_{00}/\eta_{+-} \simeq 1 - 3\varepsilon'/\eta_0, \quad (5.10)$$

[2] Note that, although the freedom associated with ξ_K and $\xi_{\ell+}-\xi_{\ell-}$ allows one to take $\alpha_K = 0$ and $\alpha_\ell = 0$ (instead of ${\rm Im}(\varepsilon_\ell) = 0$) respectively, we prefer not to do so. Note also that our parametrization (4.9a,b) is similar to, but different from the one more widely adopted [23,24],

$$\langle\ell^+|H_{\rm W}|K^0\rangle = F_\ell(1-y_\ell), \quad \langle\ell^-|H_{\rm W}|\overline{K^0}\rangle = F_\ell^*(1+y_\ell^*),$$

and that, nevertheless, our ${\rm Re}(\varepsilon_\ell)$ is exactly equivalent to $-{\rm Re}(y_\ell)$ introduced through these equations and also to $-{\rm Re}(y)$ defined in [14–16]

[3] As emphasized in [18], the classification of the symmetry-violating parameters into "direct" and "indirect" ones makes sense only when they are defined in such a way that they are invariant under rephasing of $|K^0\rangle$ and $|\overline{K^0}\rangle$, see (4.11)

[4] As a matter of fact, we have already assumed that the CP, T and CPT violations are small in deriving (4.3a) $\sim$ (4.6b) and in parametrizing the relevant amplitudes as in (4.8a) $\sim$ (4.9d)

or

$$\mathrm{Re}(\varepsilon'/\eta_0) \simeq (1/6)(1 - |\eta_{00}/\eta_{+-}|^2), \quad (5.11a)$$
$$\mathrm{Im}(\varepsilon'/\eta_0) \simeq -(1/3)\Delta\phi, \quad (5.11b)$$

where

$$\Delta\phi \equiv \phi_{00} - \phi_{+-}. \quad (5.12)$$

On the other hand, from (3.5), (5.1), (5.2) and (5.7), one may derive

$$\varepsilon'/\eta_0 = -\mathrm{i}\mathrm{Re}(\omega')(\varepsilon_2 - \varepsilon_0)\mathrm{e}^{-\mathrm{i}\Delta\phi'}/[|\eta_0|\cos(\delta_2 - \delta_0)], \quad (5.13)$$

where

$$\Delta\phi' \equiv \phi_0 - \delta_2 + \delta_0 - \pi/2. \quad (5.14)$$

Furthermore, noting that

$$\langle K_\mathrm{S}|K_\mathrm{L}\rangle \simeq 2[\mathrm{Re}(\varepsilon) - \mathrm{i}\mathrm{Im}(\delta)],$$

one may use the Bell–Steinberger relation, (2.10), to express $\mathrm{Re}(\varepsilon)$ and $\mathrm{Im}(\delta)$ in terms of measured quantities. By taking 2π, 3π, $\pi^+\pi^-\gamma$ and $\pi\ell\nu_\ell$ intermediate states into account in (2.11) and making use of the fact $\gamma_\mathrm{S} \gg \gamma_\mathrm{L}$, we derive

$$\begin{aligned}\mathrm{Re}(\varepsilon) \simeq{}& \frac{1}{\sqrt{\gamma_\mathrm{S}^2 + 4(\Delta m)^2}} \\ &\times \Big[\gamma_\mathrm{S}(\pi^+\pi^-)|\eta_{+-}|\cos(\phi_{+-} - \phi_\mathrm{SW}) \\ &+ \gamma_\mathrm{S}(\pi^0\pi^0)|\eta_{00}|\cos(\phi_{00} - \phi_\mathrm{SW}) \\ &+ \gamma_\mathrm{S}(\pi^+\pi^-\gamma)|\eta_{+-\gamma}|\cos(\phi_{+-\gamma} - \phi_\mathrm{SW}) \\ &+ \gamma_\mathrm{L}(\pi^+\pi^-\pi^0)\{\mathrm{Re}(\eta_{+-0})\cos\phi_\mathrm{SW} \\ &- \mathrm{Im}(\eta_{+-0})\sin\phi_\mathrm{SW}\} \\ &+ \gamma_\mathrm{L}(\pi^0\pi^0\pi^0)\{\mathrm{Re}(\eta_{000})\cos\phi_\mathrm{SW} \\ &- \mathrm{Im}(\eta_{000})\sin\phi_\mathrm{SW}\} \\ &+ 2\sum_\ell \gamma_\mathrm{L}(\pi\ell\nu_\ell)\{\mathrm{Re}(\varepsilon_\ell)\cos\phi_\mathrm{SW} \\ &- \mathrm{Im}(x_\ell^{(+)})\sin\phi_\mathrm{SW}\}\Big], \end{aligned} \quad (5.15)$$

$$\begin{aligned}\mathrm{Im}(\delta) \simeq{}& \frac{1}{\sqrt{\gamma_\mathrm{S}^2 + 4(\Delta m)^2}} \\ &\times [-\gamma_\mathrm{S}(\pi^+\pi^-)|\eta_{+-}|\sin(\phi_{+-} - \phi_\mathrm{SW}) \\ &- \gamma_\mathrm{S}(\pi^0\pi^0)|\eta_{00}|\sin(\phi_{00} - \phi_\mathrm{SW}) \\ &- \gamma_\mathrm{S}(\pi^+\pi^-\gamma)|\eta_{+-\gamma}|\sin(\phi_{+-\gamma} - \phi_\mathrm{SW}) \\ &+ \gamma_\mathrm{L}(\pi^+\pi^-\pi^0)\{\mathrm{Re}(\eta_{+-0})\sin\phi_\mathrm{SW} \\ &+ \mathrm{Im}(\eta_{+-0})\cos\phi_\mathrm{SW}\} \\ &+ \gamma_\mathrm{L}(\pi^0\pi^0\pi^0)\{\mathrm{Re}(\eta_{000})\sin\phi_\mathrm{SW} \\ &+ \mathrm{Im}(\eta_{000})\cos\phi_\mathrm{SW}\} \\ &+ 2\sum_\ell \gamma_\mathrm{L}(\pi\ell\nu_\ell)\{\mathrm{Re}(\varepsilon_\ell)\sin\phi_\mathrm{SW} \\ &+ \mathrm{Im}(x_\ell^{(+)})\cos\phi_\mathrm{SW}\}]. \end{aligned} \quad (5.16)$$

If, however, one retains the contribution of the 2π intermediate states alone, which is justfiable empirically, the Bell–Steinberger relation gives simply

Table 1. Input data (1)

	Quantity	Value	Unit	Ref.
	τ_S	0.896 ± 0.0007	10^{-10} s	[11]
	τ_L	5.17 ± 0.04	10^{-8} s	[17]
	$-\Delta$	0.5268 ± 0.0015	$10^{10}\mathrm{s}^{-1}$	[11]
2π	$\gamma_\mathrm{S}(\pi^+\pi^-)/\gamma_\mathrm{S}$	68.61 ± 0.28	%	[17]
	$\gamma_\mathrm{S}(\pi^0\pi^0)/\gamma_\mathrm{S}$	31.39 ± 0.28	%	[17]
	$\delta_2 - \delta_0$	(-42 ± 20)	°	[27][a]
	$\lvert\eta_{+-}\rvert$	2.285 ± 0.019	10^{-3}	[17]
	ϕ_{+-}	43.5 ± 0.6	°	[17]
	$\lvert\eta_{00}/\eta_{+-}\rvert^2$	0.9832 ± 0.0025		[11]
		0.9889 ± 0.0044		[12]
	$\Delta\phi$	0.09 ± 0.46	°	[11]
$\pi\ell\nu$	d_L^ℓ	3.27 ± 0.12	10^{-3}	[17]

[a] Error extended arbitrarily by a factor of five

$$\mathrm{Re}(\varepsilon) - \mathrm{i}\mathrm{Im}(\delta) \simeq |\eta_0|\mathrm{e}^{\mathrm{i}\Delta\phi''}\cos\phi_\mathrm{SW}, \quad (5.17)$$

where

$$\Delta\phi'' \equiv \phi_0 - \phi_\mathrm{SW}. \quad (5.18)$$

It is to be noted that equations exactly the same as (5.17) can be derived from (4.5b) and (4.6a).

6 Numerical analysis (1). Constraints from the KTeV and NA48 data

The data used as inputs in the numerical analysis given below are tabulated in Table 1. As the value of $|\eta_{00}/\eta_{+-}|$ or $\mathrm{Re}(\varepsilon'/\eta_0)$, we adopt those [11,12] reported by the KTeV Collaboration and the NA48 Collaboration[5], and as the values of Δm, τ_S and $\Delta\phi$, we use those reported by the KTeV Collaboration [11]. As for $\delta_2 - \delta_0$, we use $(-42\pm20)°$, i.e., the Chell–Olsson value [27] with the error arbitrarily extended by a factor of five to take account of its possible uncertainty [28]. All the other data are from the Particle Data Group (PDG) [17].

Our analysis consists of two parts:

The first half. We use (4.7b) to find ϕ_SW from Δm and γ_S, use (3.6) and (5.4) to find $\mathrm{Re}(\omega')$ from $\gamma_\mathrm{S}(\pi^+\pi^-)/\gamma_\mathrm{S}$ and $\gamma_\mathrm{S}(\pi^0\pi^0)/\gamma_\mathrm{S}$, and further use (5.11a,b) and (5.9) to find $\mathrm{Re}(\varepsilon'/\eta_0)$, $\mathrm{Im}(\varepsilon'/\eta_0)$, $|\eta_0|$ and ϕ_0 from $|\eta_{00}/\eta_{+-}|$, $\Delta\phi$, $|\eta_{+-}|$ and ϕ_{+-}. These results are shown as the intermediate outputs in Table 2.

The second half. The values of η_0, ε'/η_0, ϕ_SW and $\mathrm{Re}(\omega')$ obtained, supplemented with the value of $\delta_2 - \delta_0$, are used

[5] The experimental result in favor of $\varepsilon'/\varepsilon \neq 0$ was reported earlier by the NA31 Collaboration [25]. However, since another result in conflict with this result was reported almost the same time by the E731 Collaboration [26], we shall ignore both of these results. Note that ε used in [11,12] corresponds to our η_0. Since only $\mathrm{Re}(\varepsilon'/\varepsilon)$, but not $|\eta_{00}/\eta_{+-}|^2$, is reported explicitly in [11], we take a weighted average of the two values of $\mathrm{Re}(\varepsilon'/\varepsilon)$ reported in [11,12] and list this in Table 2 below

Table 2. Intermediate outputs

Quantity	Value	Unit
ϕ_{SW}	43.40 ± 0.09	$\circ$
$\mathrm{Re}(\omega')$	1.458 ± 0.157	$\times 10^{-2}$
$\mathrm{Re}(\varepsilon'/\eta_0)$	2.59 ± 0.36	$\times 10^{-3}$
$\mathrm{Im}(\varepsilon'/\eta_0)$	-0.524 ± 2.676	$\times 10^{-3}$
$\lvert\eta_0\rvert$	2.279 ± 0.019	$\times 10^{-3}$
ϕ_0	43.53 ± 0.94	$\circ$

Table 3. Constraints (in units of 10^{-3}) to CP, T and/or CPT-violating prameters (1)

Quantity	Result
$\mathrm{Re}(\varepsilon_2 - \varepsilon_0)$	0.084 ± 0.328
$\mathrm{Im}(\varepsilon_2 - \varepsilon_0)$	0.295 ± 0.113
$\mathrm{Re}(\varepsilon - \delta + \varepsilon_\ell - x_\ell^{(-)})$	1.635 ± 0.060
$\mathrm{Re}(\varepsilon)$	1.656 ± 0.014
$\mathrm{Im}(\varepsilon + \varepsilon_0)$	1.566 ± 0.014
$\mathrm{Im}(\delta)$	-0.004 ± 0.027
$\mathrm{Re}(\delta - \varepsilon_0)$	0.004 ± 0.026
$\mathrm{Re}(\delta - \varepsilon_\ell + x_\ell^{(-)})$	0.021 ± 0.062

as inputs to find $\mathrm{Re}(\varepsilon_2 - \varepsilon_0)$ and $\mathrm{Im}(\varepsilon_2 - \varepsilon_0)$ with the help of (5.13) and (5.14), and to find $\mathrm{Re}(\varepsilon)$ and $\mathrm{Im}(\delta)$ with the help of (5.17). The values of $\mathrm{Re}(\varepsilon)$ and $\mathrm{Im}(\delta)$ are in turn used to constrain $\mathrm{Re}(\delta-\varepsilon_0)$ and $\mathrm{Im}(\varepsilon+\varepsilon_0)$ through (5.2) and constrain $\mathrm{Re}(\delta - \varepsilon_\ell + x_\ell^{(-)})$ through (5.5). The numerical results obtained are shown in Table 3. The value of $\mathrm{Re}(\varepsilon - \delta + \varepsilon_\ell - x_\ell^{(-)})$, which is nothing but the value of $d_{\mathrm{L}}^\ell/2$, is also shown.

7 Numerical analysis (2). Constraints from the CPLEAR results

Immediately after the CPLEAR Collaboration reported [9] their preliminary result on the asymptotic leptonic asymmetries, $d_{1,2}^\ell(t \gg \tau_{\mathrm{S}})$, we showed [6] that this result, combined with the other relevant data available, could be used with the help of the Bell–Steinberger relation to constrain many of the CP, T and/or CPT-violating parameters introduced. The analysis went as follows. Assuming $\mathrm{Re}(x_\ell^{(-)}) = 0$[6], (5.6a) and (5.15) were used to find the values of $\mathrm{Re}(\varepsilon)$ and $\mathrm{Re}(\varepsilon_\ell)$. The value of $\mathrm{Re}(\varepsilon_\ell)$ was then used to constrain $\mathrm{Re}(\delta)$ and $\mathrm{Im}(\delta)$ through (5.6b) and (5.16), respectively, and all these values were combined with the value of η_0 to determine or constrain $\mathrm{Im}(\varepsilon+\varepsilon_0)$ and $\mathrm{Re}(\varepsilon_0)$.

In order to appreciate the results obtained under the 2π dominance and to separately constrain, as far as possible, the parameters not yet separately constrained in the previous section, we now proceed to perform an analysis similar to the one explained above [6], with the new results [13–16] reported by the CPLEAR Collaboration taken into account. In [14–16], the CPLEAR Collaboration has defined two kinds of experimental asymmetries $A_T^{\mathrm{exp}}(t)$ and $A_\delta^{\mathrm{exp}}(t)$ which are related to $d_{1,2}^\ell(t)$ and behave as

$$A_T^{\mathrm{exp}}(t \gg \tau_{\mathrm{S}}) \simeq 4\mathrm{Re}(\varepsilon + \varepsilon_\ell - x_\ell^{(-)}), \qquad (7.1a)$$

$$A_\delta^{\mathrm{exp}}(t \gg \tau_{\mathrm{S}}) \simeq 8\mathrm{Re}(\delta), \qquad (7.1b)$$

and, by performing

(1) a fit to A_T^{exp} under the assumption of $\mathrm{Re}(\varepsilon_\ell) = 0$ and $x_\ell^{(-)} = 0$ [14],
(2) a fit to A_δ^{exp} [15], and
(3) fit to both A_T^{exp} and A_δ^{exp} using as constraints the Bell–Steinberger relation and the PDG value of d_{L}^ℓ [16], succeeded in determining $\mathrm{Re}(\varepsilon)$, $\mathrm{Re}(\delta)$, $\mathrm{Im}(\delta)$, $\mathrm{Re}(\varepsilon_\ell)$, $\mathrm{Im}(x_\ell^{(+)})$ and/or $\mathrm{Re}(x_\ell^{(-)})$ simultaneously.

Among the numerical outputs obtained by the CPLEAR Collaboration, there are two pieces, $\mathrm{Re}(\varepsilon) = (1.55 \pm 0.35) \times 10^{-3}$ from [14] and $\mathrm{Re}(\delta) \simeq (0.30 \pm 0.33) \times 10^{-3}$ from [15], which are in fact determined predominantly by the asymptotic values of $A_T^{\mathrm{exp}}(t)$ and $A_\delta^{\mathrm{exp}}(t)$[7]. One may therefore interpret these outputs as giving the values of $A_T^{\mathrm{exp}}(t \gg \tau_{\mathrm{S}})/4$ and $A_\delta^{\mathrm{exp}}(t \gg \tau_{\mathrm{S}})/8$, respectively. Replacing (5.6a) with (7.1a), using (5.5) instead of (5.6b), and with the data listed in Table 4 as well as in Table 1 used as inputs, we perform an analysis similar to the previous one [6], and obtain the result shown in Table 5.

A couple of remarks are in order.

(1) The assumption of $x_\ell^{(-)} = 0$ has little influence numerically on the determination of $\mathrm{Re}(\varepsilon)$, $\mathrm{Im}(\delta)$ and $\mathrm{Im}(\varepsilon+\varepsilon_0)$ and the error of these parameters is dominated by that of η_{000}.
(2) Our constraint to $\mathrm{Re}(\varepsilon_\ell)$ is better to be interpreted as a constraint to $\mathrm{Re}(\varepsilon_\ell - x_\ell^{(-)})$, the error of which is controlled dominantly by that of A_T^{exp}.
(3) The error of $\mathrm{Re}(\delta)$ and $\mathrm{Re}(\varepsilon_0)$ is also controlled dominantly by that of A_T^{exp}.
(4) The numerical results we have obtained are fairly in agreement with those obtained by the CPLEAR Collaboration in [16], except that we have not been able to separate $\mathrm{Re}(\varepsilon_\ell)$ from $\mathrm{Re}(x_\ell^{(-)})$.

8 Case study: T or CPT violation?

In the analyses given in the previous sections, we have taken account of the possibility that any of CP, T and CPT symmetries might be violated in the K^0–$\overline{K^0}$ system. Our numerical results shown in Table 3 and Table 5

[6] In most of the experimental analyses prior to those [15, 16] by the CPLEAR Collaboration, either CPT symmetry is taken as granted or no distinction is made between $x_{\ell+}$ and $x_{\ell-}$, which implies that $x_\ell^{(-)}$ is implicitly presupposed to be zero. Accordingly, we identified x used in [17] with our $x_\ell^{(+)}$

[7] In contrast, the values of $\mathrm{Im}(x_\ell^{(+)})$, $\mathrm{Re}(x_\ell^{(-)})$ and $\mathrm{Im}(\delta)$ obtained are sensitive to the behavior of $A_T^{\mathrm{exp}}(t)$ and of $A_\delta^{\mathrm{exp}}(t)$ at t comparable to τ_{S}

Table 4. Input data (2)

	Quantity	Value	Unit	Ref.
3π	$\gamma_L(\pi^+\pi^-\pi^0)/\gamma_L$	12.56 ± 0.20	%	[17]
	$\gamma_L(\pi^0\pi^0\pi^0)/\gamma_L$	21.12 ± 0.27	%	[17]
	$\mathrm{Re}(\eta_{+-0})$	-0.002 ± 0.008		[13,16]
	$\mathrm{Im}(\eta_{+-0})$	-0.002 ± 0.009		[13,16]
	$\mathrm{Re}(\eta_{000})$	0.08 ± 0.11		[13,16]
	$\mathrm{Im}(\eta_{000})$	0.07 ± 0.16		[13,16]
$\pi^+\pi^-\gamma$	$\gamma_S(\pi^+\pi^-\gamma)/\gamma_S$	0.178 ± 0.005	%	[17]
	$\lvert\eta_{+-\gamma}\rvert$	2.35 ± 0.07	10^{-3}	[17]
	$\phi_{+-\gamma}$	44 ± 4	°	[17]
$\pi\ell\nu$	$\sum_\ell \gamma_L(\pi\ell\nu)/\gamma_L$	65.96 ± 0.30	%	[17]
	$\mathrm{Im}(x_\ell^{(+)})$	-0.003 ± 0.026		[17]
	$A_T^{\exp}(t \gg \tau_S)$	6.2 ± 1.4	10^{-3}	[14]

Table 5. Constraints (in units of 10^{-3}) to CP, T and/or CPT-violating prameters (2)

Quantity	Result
$\mathrm{Re}(\varepsilon)$	1.666 ± 0.048
$\mathrm{Im}(\varepsilon+\varepsilon_0)$	1.590 ± 0.059
$\mathrm{Im}(\delta)$	0.020 ± 0.051
$\mathrm{Re}(\delta)$	-0.085 ± 0.361
$\mathrm{Re}(\varepsilon_0)$	-0.099 ± 0.365
$\mathrm{Re}(\varepsilon_\ell)$	-0.116 ± 0.353

indicate that the CPT symmetry appears to be consistent with the experiments while T symmetry does not appear to be consistent with experiments[8]. To confirm these observations, we now go on to perform a case study.

8.1 Case A. CPT is a good symmetry

Putting

$$\delta = \mathrm{Re}(\varepsilon_I) = \mathrm{Re}(\varepsilon_\ell) = x_\ell^{(-)} = 0,$$

(5.2), (5.5), (5.6a,b), (5.8) and (5.13) reduce, respectively, to

$$\eta_I \simeq \varepsilon + \mathrm{iIm}(\varepsilon_I), \tag{8.1a}$$
$$d_L^\ell \simeq 2\mathrm{Re}(\varepsilon), \tag{8.1b}$$
$$d_1^\ell(t \gg \tau_S) \simeq 4\mathrm{Re}(\varepsilon), \tag{8.1c}$$
$$d_2^\ell(t \gg \tau_S) \simeq 0, \tag{8.1d}$$
$$d_L^\ell \simeq d_1^\ell(t \gg \tau_S)/2, \tag{8.1e}$$
$$\varepsilon'/\eta_0 \simeq \mathrm{Re}(\omega')\mathrm{Im}(\varepsilon_2-\varepsilon_0)\mathrm{e}^{-\mathrm{i}\Delta\phi'} /[|\eta_0|\cos(\delta_2-\delta_0)]. \tag{8.1f}$$

Equation (8.1f) gives[9]

$$\mathrm{Im}(\varepsilon'/\eta_0) = -\mathrm{Re}(\varepsilon'/\eta_0)\tan\Delta\phi', \tag{8.2}$$

and the simplified version of the Bell–Steinberger relation, (5.17), gives[10]

$$\phi_0 \simeq \phi_{SW}, \tag{8.3a}$$
$$\mathrm{Re}(\varepsilon) \simeq |\eta_0|\cos\phi_{SW}. \tag{8.3b}$$

From the input data (Table 1 and Table 4) and the intermediate output data (Table 2), we observe the following:
(1) The experimental values of d_L^ℓ, $d_1^\ell(t \gg \tau_S)$ and $d_2^\ell(t \gg \tau_S)$ are compatible with (8.1d,e).
(2) The values of $\mathrm{Re}(\varepsilon'/\eta_0)$, $\mathrm{Im}(\varepsilon'/\eta_0)$, ϕ_0 and $\delta_2-\delta_0$ are, as illustrated in Fig. 1, compatible with (8.2).
(3) The values of ϕ_0 and ϕ_{SW} are compatible with (8.3a).
(4) The values of $\mathrm{Re}(\varepsilon)$ determined from (8.1a) and (8.1b), $(1.652 \pm 0.029) \times 10^{-3}$ and $(1.635 \pm 0.060) \times 10^{-3}$, are compatible with each other and, as a weighted average, give

$$\mathrm{Re}(\varepsilon) \simeq (1.649 \pm 0.026) \times 10^{-3}, \tag{8.4}$$

which is compatible with $(1.656\pm0.014)\times10^{-3}$ determined with the aid of the Bell–Steinberger relation (8.3b)[11].
(5) Equations (8.1a) and (8.1f) give

$$\mathrm{Im}(\varepsilon+\varepsilon_0) \simeq (1.570 \pm 0.030) \times 10^{-3}, \tag{8.5a}$$
$$\mathrm{Im}(\varepsilon_2-\varepsilon_0) \simeq (3.02 \pm 1.09) \times 10^{-4}. \tag{8.5b}$$

8.2 Case B. T is a good symmetry[12]

Putting

$$\varepsilon = \mathrm{Im}(\varepsilon_I) = \mathrm{Im}(x_\ell^{(+)}) = \mathrm{Im}(x_\ell^{(-)}) = 0,$$

[8] Our results should also be compared with earlier results obtained by other authors from analyses more or less similar to ours. Although our analysis is much more comprehensive and our results are much more precise than these earlier ones, a conclusion similar to ours, i.e., the observed CP violation is predominantly due to the CPT-conserving parameters and time reversal invariance is violated, has already been reached, for example, by Schubert et al. [29]

[9] It is to be noted that, if and only if the CPT symmetry is supplemented with the very accidental empirical fact $\phi_{SW} \simeq \delta_2-\delta_0+\pi/2$, one would have $\mathrm{Im}(\varepsilon'/\eta_0) \simeq 0$; it is therefore, as emphasized in [7], not adequate to assume this in a phenomenological analysis

[10] Equation (8.3a) states that the deviation of ϕ_0 from ϕ_{SW} measures CPT violation. This is equivalent to the more familiar statement: deviation of $(2/3)\phi_{+-}+(1/3)\phi_{00}$ from ϕ_{SW} measures CPT violation, because (5.9), supplemented with the experimental observation $|\eta_{00}/\eta_{+-}| \simeq 1$ and $\Delta\phi \simeq 0$, gives $\phi_0 \simeq (2/3)\phi_{+-}+(1/3)\phi_{00}$

[11] Equation (5.15), with $\mathrm{Re}(\varepsilon_\ell) = 0$, yields $(1.667 \pm 0.048) \times 10^{-3}$

[12] The possibility of CP/CPT violation in the framework of T symmetry was examined before by one of the present authors (S.Y.T) [30] when the experimental results which upset the CPT symmetry (e.g., $|\boldsymbol{\eta}_{+-}|$ is nearely twice as large as $|\boldsymbol{\eta}_{+-}|$!) had been reported. The same possibility was recently reconsidered by Bigi and Sanda [24]

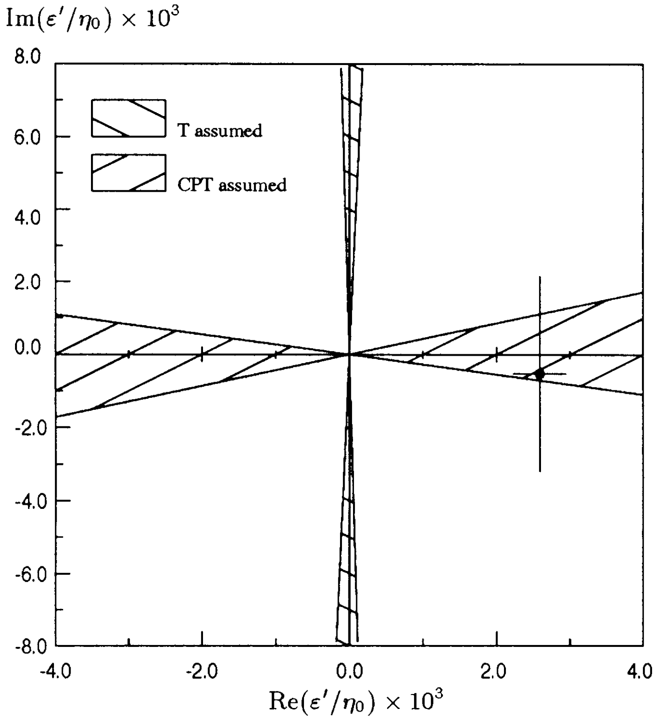

Fig. 1. The allowed region on the complex ε'/η_0 plane when CPT or T symmetry is assumed. (The experimental value of ε'/η_0 is also shown)

(5.2), (5.5), (5.6a) and (5.13) reduce, respectively, to

$$\eta_I \simeq -\delta + \mathrm{Re}(\varepsilon_I), \tag{8.6a}$$

$$d^\ell_{\mathrm{L}} \simeq -2\mathrm{Re}(\delta) + 2\mathrm{Re}(\varepsilon_\ell - x^{(-)}_\ell), \tag{8.6b}$$

$$d^\ell_1(t \gg \tau_{\mathrm{S}}) \simeq 2\mathrm{Re}(\varepsilon_\ell - x^{(-)}_\ell), \tag{8.6c}$$

$$\varepsilon'/\eta_0 \simeq \mathrm{Re}(\omega')\mathrm{Re}(\varepsilon_2 - \varepsilon_0)\mathrm{e}^{-\mathrm{i}(\Delta\phi' + \pi/2)} /[|\eta_0| \cos(\delta_2 - \delta_0)]. \tag{8.6d}$$

Equation (8.6d) gives

$$\mathrm{Im}(\varepsilon'/\eta_0) = \mathrm{Re}(\varepsilon'/\eta_0) \cot \Delta\phi', \tag{8.7}$$

and the simplified version of the Bell–Steinberger relation, (5.17), gives

$$\phi_0 \simeq \phi_{\mathrm{SW}} \pm \pi/2, \tag{8.8a}$$

$$\mathrm{Im}(\delta) \simeq \pm|\eta_0| \cos\phi_{\mathrm{SW}}. \tag{8.8b}$$

From the input data (Table 1 and Table 4) and the intermediate output data (Table 2), we observe the following:

(1) As illustrated also in Fig.1, the values of $\mathrm{Re}(\varepsilon'/\eta_0)$, $\mathrm{Im}(\varepsilon'/\eta_0)$, ϕ_0 and $\delta_2 - \delta_0$ are not compatible with (8.7).
(2) The values of ϕ_0 and ϕ_{SW} are not compatible with (8.8a).
(3) Equation (8.6a) gives

$$\mathrm{Im}(\delta) \simeq (-1.570 \pm 0.030) \times 10^{-3}, \tag{8.9}$$

to be compared with $\pm(1.656 \pm 0.014) \times 10^{-3}$ determined with the aid of (8.8b).

(4) Equation (8.6d) gives

$$\mathrm{Re}(\varepsilon_2 - \varepsilon_0) \simeq (0.61 \pm 3.12) \times 10^{-4}, \tag{8.10}$$

while (8.6a,b,c) give in turn

$$\mathrm{Re}(\varepsilon_\ell - x^{(-)}_\ell) \simeq (3.14 \pm 1.40) \times 10^{-3}, \tag{8.11a}$$

$$\mathrm{Re}(\delta) \simeq (1.51 \pm 1.40) \times 10^{-3}, \tag{8.11b}$$

$$\mathrm{Re}(\varepsilon_0) \simeq (3.16 \pm 1.40) \times 10^{-3}. \tag{8.11c}$$

The observation (1) establishes the existence of direct CP/T violation in the K^0–$\overline{K^0}$ system [11,12,31][13]. The observation (2), though subject to the validity of the Bell–Steinberger relation, also implies that CP/T symmetry is violated in the K^0–$\overline{K^0}$ system.

9 Summary and concluding remarks

In order to identify or search for the violation of the CP, T and CPT symmetries in the K^0–$\overline{K^0}$ system, parametrizing the mixing parameters and the relevant decay amplitudes in a convenient and well-defined way, we have, with the aid of the Bell–Steinberger relation and with all the relevant experimental data used as inputs, performed numerical analyses to derive constraints to the symmetry-violating parameters in several ways. The analysis given in Sect. 6 is based on the data on 2π decays as well as the well-measured leptonic asymmetry d^ℓ_{L}, while, in the analysis given in Sect. 7, the data on 3π and $\pi^+\pi^-\gamma$ decays and on the newly measured leptonic asymmetries are also taken into account.

The numerical outputs of our analyses are shown in Table 3 and Table 5, and the main results may be summarized as follows:

(1) The 2π data directly give $\mathrm{Im}(\varepsilon_2 - \varepsilon_0) = (2.95 \pm 1.13) \times 10^{-4}$ in general, or $(3.02 \pm 1.09) \times 10^{-4}$ if the CPT symmetry is assumed, where a possible large uncertainty associated with $\delta_2 - \delta_0$ has been fully taken into account. This result indicates that the CP and T symmetries are definitively violated in the decays of K^0 and $\overline{K^0}$ into the 2π states.
(2) The well-measured leptonic asymmetry d^ℓ_{L} directly gives $\mathrm{Re}(\varepsilon - \delta + \varepsilon_\ell - x^{(-)}_\ell) = (1.635 \pm 0.060) \times 10^{-3}$, which implies presumably that the CP and T violations are present also in the K^0–$\overline{K^0}$ mixing (i.e., $\mathrm{Re}(\varepsilon) \neq 0$)[14]
(3) The Bell–Steinberger relation, with the 2π intermediate states alone taken into account, gives $\mathrm{Re}(\varepsilon) = (1.656 \pm 0.014) \times 10^{-3}$ and $\mathrm{Im}(\varepsilon + \varepsilon_0) = (1.566 \pm 0.014) \times 10^{-3}$. If the CPT symmetry is assumed, $\mathrm{Re}(\varepsilon)$ is determined without recourse to the Bell–Steinberger relation to be $(1.649 \pm 0.026) \times 10^{-3}$. All this indicates that the CP and T violations are present in the mixing parameters.

[13] We like to mention that (8.7) would become consistent with the experiments if, say, ϕ_{00} would prove to be away from ϕ_{+-} roughly by $\simeq 6°$ or more

[14] Of course, $d^\ell_{\mathrm{L}} \neq 0$ does not exclude CPT violation (i.e., $\mathrm{Re}(\delta - \varepsilon_\ell + x^{(-)}_\ell) \neq 0$)

(4) The parameters, the nonvanishing of which signals the CP and CPT violations, have also been constrained. The 2π data alone directly give $\mathrm{Re}(\varepsilon_2-\varepsilon_0) = (0.084\pm0.328)\times 10^{-3}$ and, with the aid of the Bell–Steinberger relation, give $\mathrm{Im}(\delta) = (-0.004\pm0.027)\times10^{-3}$, $\mathrm{Re}(\delta-\varepsilon_0) = (0.004\pm 0.026)\times10^{-3}$ and $\mathrm{Re}(\delta-\varepsilon_\ell+x_\ell^{(-)}) = (0.021\pm0.062)\times10^{-3}$. These results imply that there is no evidence for CPT violation on the one hand and that CPT symmetry is tested at best to the level of a few $\times10^{-5}$ on the other hand.
(5) The Bell–Steinberger relation, even with the intermediate states other than the 2π states taken into account, still allows one to determine $\mathrm{Re}(\varepsilon)$ and $\mathrm{Im}(\varepsilon+\varepsilon_0)$ and to constrain $\mathrm{Im}(\delta)$ to a level better than 10^{-4}. On the other hand, the constraint to $\mathrm{Re}(\delta)$, $\mathrm{Re}(\varepsilon_0)$ and $\mathrm{Re}(\varepsilon_\ell - x_\ell^{(-)})$ is a little loose and is at the level of a few $\times10^{-4}$.

The recent data reported by the KTeV Collabotration [11] and the NA48 Collaboration [12] are extremely remarkable in that they play a vital role in establishing $\mathrm{Im}(\varepsilon_2-\varepsilon_0)\neq 0$, and that this is at present the only piece which indicates "direct violation" (in the sense defined in Sect. 4) of the CP and T symmetries and thereby unambiguously rules out superweak (or superweak-like) models of CP violation.

The analyses done by the CPLEAR Collaboration [14–16] are also very remarkable, in particular in that they have succeeded in deriving a constraint to $\mathrm{Re}(x_\ell^{(-)})$, and in that they have determined $\mathrm{Re}(\varepsilon+\varepsilon_\ell-x_\ell^{(-)})$ and $\mathrm{Re}(\delta)$ directly (i.e., without invoking the Bell–Steinberger relation) with an accuracy down to the level of a few $\times10^{-4}$.[15]

It is expected that the new experiments at the facilities such as DAΦNE, Frascati, will be providing data with such precision and quality that a more precise and thorough test of the CP, T and CPT symmetries, and a test of the Bell–Steinberger relation as well, become possible [23,33, 34].

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